A primer on electrical units in the Système International

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I examine the dissemination of the electrical units, from basic physical laws to commercial calibrations. I discuss the important distinction between realization and representation of units, which refers back to the distinction between SI (Le Système International d'Unités) units and practical units. I then outline the current dissemination of electrical units, heavily based on the fundamental quantum standards (Josephson voltage and quantized Hall resistance standards), as well as on a classic metrology experiment, the calculable capacitor. We will see that this chain of unit realizations is one way physicists test the accuracy of the basic physical laws. As an example of the dissemination, I also outline the transfer chain from the primary voltage standard to the factory floor. Finally, I briefly discuss some areas of current research which have the potential to induce permanent changes in the definition of SI units (in particular the kilogram), and to close the "metrology triangle."

I. MOTIVATION

I have written this primer on the dissemination of electrical units within the SI (Le Système International d'Unités) for several reasons. The primary one is that students are often quite intrigued by how the basic definitions of physical laws are translated into everyday definitions of physical units. I hope that this article provides a simple introduction into the subject for teachers, so that they can provide some context for their students.

The secondary reasons for this primer include providing a simple entrée to the field for new practitioners. Also, electrical metrology (metrology is the study of measurements) is currently evolving, and it is useful to remind workers of the significance of new possible standards [the watt balance experiment and single electron tunneling (SET) devices], and of how they may fit into the current framework.

II. LEGAL MATTERS, PHILOSOPHY OF THE SI, AND DEFINITIONS

A. Legal structure and philosophy of the SI

The legal basis for the international system of physical units is the Convention of the Meter, a treaty which originated in 1875, and which now has about 50 signatories. The SI was adopted by the Convention of the Meter in the post-World War II era (1960). Figure 1 shows the supporting legal structure of the Convention, with the diplomatic and technical areas. The CGPM (General Conference of Weights and Measures) is the diplomatic body, with ambassadors from some of the signatories. The CIPM (International Committee of Weights and Measures) is the top-level technical body, and is composed of members from major national standards laboratories and others. The BIPM (International Bureau of Weights and Measures), located in Paris, France,

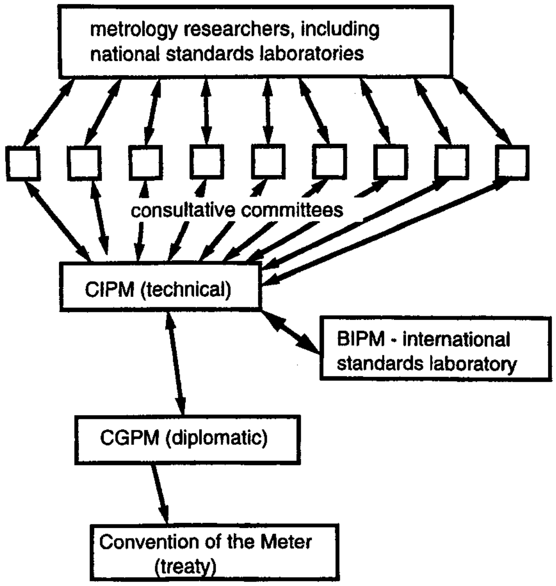

Fig. 1. The framework for technical information flow within the Meter Convention; the legal support for each level depends on the level below it.

is the *international* standards laboratory; the staff includes workers from many of the national laboratories.

The nine consultative committees provide technical advice on the SI, specifically in terms of the experiments best suited to provide standards. Proposals to change the SI definitions and representations generally originate with workers in the metrology community, and are formalized through the consultative committees; in general, the higher-level CIPM and CGPM ratify the committees' proposals.

Regarding specifically the United States, NIST (National Institute of Standards and Technology) is the national standards laboratory for the U.S. Formal participation in the international body includes the presence of a senior manager on the CIPM, and of senior managers and senior scientists on the consultative committees. For example, the U.S. representative to the Consultative Committee on Electricity (CCE) is the Deputy Director of NIST's Electronics and Electrical Engineering Laboratory. These managers are in frequent contact with the NIST researchers working in the various metrological areas.

The CCE often forms working groups, which are usually the organizations which actually originate the detailed recommendations. For example, in 1987 two working groups recommended numerical values for the constants which allow the Josephson voltage (JV) and quantized Hall resistance (QHR) experiments to be used as voltage and resistance standards, respectively; these recommended values were fixed by international agreement, and went into effect on January 1, 1990. See below for much more discussion of these constants.

The SI has two main, interdependent goals (see Ref. 1 for a list of relevant CGPM and CIPM pronouncements): (i) international agreement on a system of units for physical measurements, and (ii) the formation of a *coherent* system of units. In this context, the term "coherent" has a specific meaning: any unit should be related to any other unit by only a multiplicative and divisive combination, with a numerical prefactor of unity. For example the volt is $1\ \mathrm{V} = 1\ \mathrm{kg\ m^2/(A\ s^3)}$. Although this seems a perfectly natural idea for a system of units, we will see that it imposes stringent requirements.

B. Definitions

The coherency of the SI system brings us to an important distinction: "SI units" versus "practical units." A *SI unit* is a unit as defined by the SI, which maintains coherency. A *practical unit* is one that can be maintained by a convenient experiment that provides a useful standard for everyday use.

In turn, these two kinds of units imply two distinct classes of dissemination of the units:[2] the *realization* versus the *representation* of a unit. The generic definition of the *realization* of a unit is a physical experiment or artifact, based on well-established principles (e.g., Newton's or Maxwell's laws), that produces the unit in terms of the SI definition (the SI unit). These experiments are typically difficult, time consuming, and slow (sometimes decades long) to produce results. The generic definition of the *representation* of a unit is an experiment or artifact which produces a quantity which can be routinely compared to other standards. The routine nature of a representation allows us to "flywheel" the practical unit, i.e., to disseminate from one primary standard[3] to a large number of secondary standards. Routine calibrations in the national standards labs are generally representations, not realizations.

As an example of the distinction between realization and representation, the *representation of the volt* at NIST is the voltage output of a set of Josephson junctions as a primary standard; the JV standard routinely provides reproducibility, over time and between different labs, with a relative uncertainty of less than 10^{-8}. This reproducibility and the convenience of operation of the JV experiment make the output of the JV standard a practical unit, as defined above. This voltage output is then flywheeled at NIST and by customers using both standard cells (electrochemical cells at about 1.018 V), and solid-state electronic voltage standards which are usually based on Zener diodes (I will colloquially call these "Zeners"). Standard cells have a smaller achievable uncertainty, but Zeners are less expensive to operate, and undergo transportation better.

However, the *realization of the volt* is a completely different matter; as for most of the electrical units, the realization flows through the equivalence of electrical and mechanical energy or power. For the volt, one realization is from a force balance,[4] which uses a voltage across a capacitor to balance a gravitational force. This is a complicated and slow experiment, which cannot be used as the first step in a routine flywheel for calibrations.

The seven "base units" of the SI are the meter [m], kilogram [kg], second [s], ampere [A], kelvin [K], mole [mol], and candela [cd]. The significance of these is that all of the other units ("derived units," such as volt and newton) can be expressed in terms of the seven base units (I note briefly that the choice of the base units is somewhat arbitrary[5]—see below). For electrical units, only the first four base units are necessary.

Of the first three base units, two (meter and second) are defined as what are sometimes variously called "atomic," "fundamental," "quantum," or "natural" units; in this paper, I will use the term "fundamental unit." What this term means is that the realization of the meter and the second are accomplished by using radiation from atomic transitions; we thus believe that these units should be the same for all time, and in all places.[6] Such fundamental units are clearly preferable to those based on macroscopic experiments or bulk materials properties ("artifacts"), which may not stay constant. For example, the kilogram is not a fundamental unit (it is instead defined as the mass of a particular metal cylinder kept in Paris—the international prototype of the kilogram), and since the realization of the ampere requires the kilogram, meter, and the second, this leads to the fact that the realizations of the electrical units are also based on macroscopic experiments, even though higher-reproducibility fundamental standards based on quantum phenomena such as the JV and the QHR experiments are available. We will see that there are experiments in progress which may lead to changes in the definition of the kilogram, so as to allow fundamental realizations of the electrical units (and others). For a much longer exposition of this general topic, see Ref. 6.

As an aside: the realization of the meter is now (since 1983) not independent of the realizations of the other base units, since length is realized by the pathlength traveled by light in a specified amount of time. This has the effect of defining the speed of light as a fixed quantity, and thus also the product $\mu_0\epsilon_0 = 1/c^2$ (μ_0 and ϵ_0 are the permeability and permittivity of free space).

III. REALIZATION OF THE ELECTRICAL UNITS

The base electrical unit, the ampere, was defined in 1960: "The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length."[1] Given that the equation underlying this definition is $F/L = \mu_0 I^2/(2\pi d)$, this definition has the effect of assigning to μ_0 the exact value of $4\pi\times10^{-7}$ N/A^2. Here, F/L is the force per unit length, I is the current, and d is the separation. Because the speed of light is fixed, as noted above, this also fixes ϵ_0 as a defined quantity (no uncertainty).

In contrast to what one might expect from the 1960 definition, the most commonly used practical electrical units (and the most-often calibrated) are the volt (unit of voltage [V]) and the ohm (unit of resistance [Ω]). This fact has a simple motivation: It is much easier to store and compare voltages or resistances than currents. I am also going to discuss only the time-invariant (dc) units; sinusoidal voltages, etc., are based on the dc values.

A. Framework for dissemination (representation and realization) of electrical units

It turns out that realizations for the farad and the watt can be done with much lower uncertainty than the realization of the ampere. This is the main reason why, in fact, the realization as well as the representation of the electrical units does not start with the ampere. The uncertainty of the representation is also substantially reduced by the availability of the quantum standards based on the Josephson and quantized Hall effects. These standards are much better than the previous artifactual representations (standard cells and wire-wound resistors), mainly in terms of their stability in time, their reproducibility across different national and commercial labs (thus allowing calibrations to be performed in the field, without transportation of standards to NIST), as well as the fact that they are quantum-mechanical standards based on well-established physical laws.

In fact, these practical quantum standards have better stability and reproducibility than the realizations of the SI units. Because of this, the world metrology community desired to disseminate the electrical units using these practical standards in a consistent way for all countries. The Convention of the Meter did this on January 1, 1990, by announcing accepted values for the constants $K_{J\text{-}90}$ and $R_{K\text{-}90}$, which fixed the accepted ratios between the JV standard and the SI volt,[7] and between the QHR standard and the SI ohm. Before this time, in contrast, many countries disseminated the electrical units using different primary standards, requiring the definition of practical units such as the "NBS volt" (i.e., the NIST-accepted ratio between the JV standard and the SI volt). In fact, the relations[8] of the various countries' practical volts to the SI volt varied by as much as 10 μV for a measurement of 1 V (i.e., a relative difference of 10^{-5})! In effect, the accepted values for the ratios allow the representation of the volt and the ohm (and thus most of the other electrical calibrations) to be more closely tied to the realization experiments than alternatives. For example, the output of electrochemical cells vary from cell to cell, and drift in time; thus the representation of the volt using cells would not be as closely tied to the SI volt as that using the JV standard, and would require frequent comparisons to the underlying realization.

I stress that, although the Josephson and quantized Hall effects are based on simple formulas involving only fundamental constants and (in the former) a measured frequency, the values of $K_{J\text{-}90}$ and $R_{K\text{-}90}$ are not primarily based on those formulas. Rather, the values are mainly based on experiments relating back to the SI mechanical units (kg, m, s), through the more difficult and time-consuming realization experiments.

Thus the actual realizations and subsequent representations of the electrical units are as schematically indicated in Fig. 2. Note that this framework is not an exhaustive list of the different types of realization experiments. This complication is to some extent "hidden" in the single values $K_{J\text{-}90}$ and $R_{K\text{-}90}$, whose determination in 1990 depended on results from a variety of experiments, of which the main ones are shown. The darkness of the arrows roughly reflects the importance of the various pathways for $K_{J\text{-}90}$ and $R_{K\text{-}90}$.

Briefly (detailed descriptions follow), the farad F (unit of capacitance C), is realized by the calculable capacitor experiment; this provides an absolute determination of C in terms of length L only. The value of the farad, along with the equivalence of mechanical and electrical power as determined by moving coil-type experiments, mostly determined the value of $K_{J\text{-}90}$, which is now used with JV standards for the representation of the volt. In addition the farad was used, along with a calculation using a value for the fine-structure constant α mostly derived from atomic physics experiments, to realize a value of $R_{K\text{-}90}$, and thus with QHR standards the representation of the ohm. Finally, the practical volt and the ohm, as represented through the JV and QHR standards, determine the other electrical units, including the ampere, as well as magnetic and electric fields, etc.

As we noted earlier, the choice of the base units is historical, and not unique, although the *number* of base units is constrained by the known physical laws[5] (in particular, three

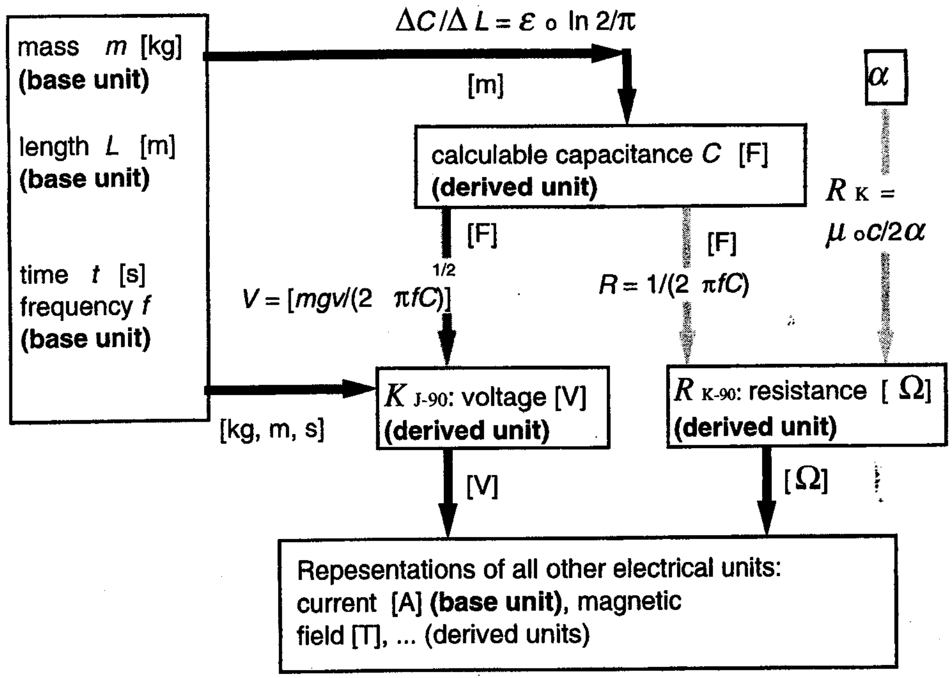

Fig. 2. Schematic framework for the representation of some of the electrical units in the SI, and the underlying realizations. Representations of the base units ([kg], [m], [s], [A]) and derived units ([F], [V], [Ω]) are indicated. The arrows and accompanying text indicate schematically the realizations. The units in square brackets near the arrows show which units are used in each of the realizations. The darkness of the arrows schematically indicate the weight of each pathway, for weighted averages.

mechanical and one electrical are required). The historical choices have been based on the most convenient definitions available to experimenters at various times over the last 150 years. In this context, we again note that the ampere, which we might expect to be the beginning of the dissemination since it is the base electrical SI unit, appears in Fig. 2 as if it were a derived unit based on several others. For example, we will see in the next section that the calculable capacitor experiment realizes the farad directly from the meter, using only electrostatics (Coulomb's law).

How is it possible, for example, for the farad to be realized independently of the ampere, given the SI definition? It is only possible because the framework for dissemination of the electrical units contains one very important (but usually unstated) assumption: that the physical laws for mechanics (e.g., equivalence of inertial and gravitational mass) and for electromagnetism (e.g., Coulomb's law) are correct, without any uncertainty. This is why metrologists can use Coulomb's law (including the defined value of ϵ_0) to realize the farad, and not use the SI definition, which flows from Ampere's law. In essence, the fact that this framework is successful (i.e., the extent to which various paths to realize the units agree) is one way that physicists confirm the basic physical laws for mechanics and electromagnetism, at least to the same levels of uncertainty that the various realizations achieve.

B. Realization of the farad via the calculable capacitor

The principle of the calculable capacitor is actually quite simple,[4] although the practice requires a highly dedicated metrologist working full time. A major difficulty, in general, with defining an absolute capacitance is the problem associated with the fringing fields; in simple terms, one cannot use the parallel-plate formula, $C=\epsilon_0 A/d$ (where A and d are plate area and separation) unless the plates are of infinite area, because of the fringing fields that curve around the edges. Of course, one could, in principle, numerically calculate the capacitance of any arbitrary shape. However, the difficulty is that it is impossible in practice to align and measure the sizes and separations accurately enough to achieve desired relative uncertainties (which for the electrical units are typically 10^{-8}).

The elegance of the calculable capacitor standard (I specialize to the one at NIST) flows from the fact that it does not pursue such a "brute force" approach. Instead, the success of the calculable capacitor is based on the observation[9] that a particular geometry rejects the effect of the fringing fields; in particular, to first order, *the capacitance depends on only one length.* As currently used at NIST,[10] this geometry consists of four long cylinders at the vertices of a square, with the capacitance measured between either (nominally identical) pair of opposite electrodes (Fig. 3). A fifth mobile electrode at the center of the square is then displaced, and the measured capacitance obeys

$$\Delta C/\Delta L=\epsilon_0 \ln 2/\pi,$$

where ΔL is the displacement. As noted above, ϵ_0 is now (as of 1983) a defined quantity, and so imparts no uncertainty in this relationship. Thus this experiment forms a direct realization of the SI farad based only on the realization of the meter and the value of ϵ_0 (although this is certainly not the realization contained in the definition of the ampere). It also has the great advantage of depending on a measurement of displacement, rather than the more difficult measurement of absolute length. The measurement corresponds to about 2 pF per meter of displacement; NIST's version measures a change of 0.5 pF.

Thus we see that the calculable capacitor can realize the SI farad from the SI meter. As indicated in Fig. 2 and discussed in the preceding text, from this capacitance one can deter-

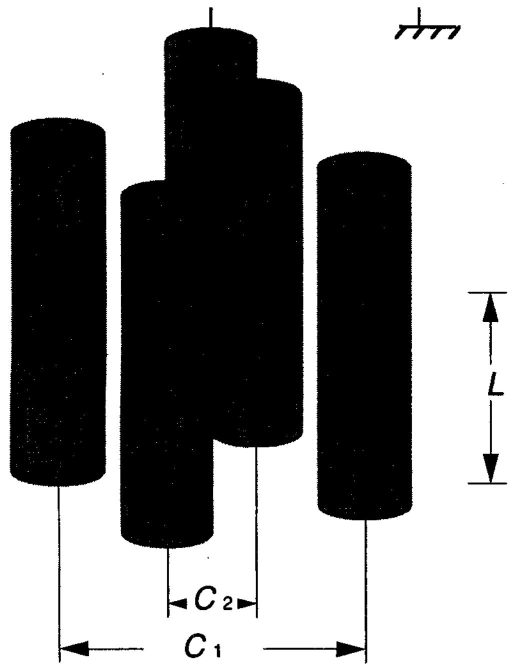

Fig. 3. Schematic drawing of the calculable capacitor, showing the four fixed electrodes, and the fifth mobile, grounded electrode in the middle. The capacitance is measured between either opposite pair of fixed electrodes. More precisely, one grounds three of the four cylinders that form the square, applies a voltage to the fourth cylinder, and then measures the ratio of that voltage to the charge on the diagonally opposite (grounded) cylinder. For more information, see Petley, Ref. 6, pp. 142–145 and references cited therein.

mine an impedance or resistance. Most other electrical realizations (such as the watt balance) depend on equivalence of force, energy, or power, and thus generally realize a combination of electrical units. Thus they require the output from the calculable capacitor to separate out the other units, such as voltage and resistance. We now turn to the realization and representation of these two latter units, which involve the JV and QHR standards.

C. Quantum standards: General themes for the assignment of the values for $K_{J\text{-}90}$ and $R_{K\text{-}90}$

The Working Group on the Josephson Effect reported to the CCE in August 1988 on its recommendation for $K_{J\text{-}90}$.[11] The theme of this recommendation (true also for $R_{K\text{-}90}$) was: "the following guiding principle is adopted for their derivation: The values should be so chosen that they are unlikely to require significant change in the foreseeable future. This means that the number of digits given for the recommended values should be the minimum possible and that the uncertainties should be conservatively assigned."[11] It is clear from the descriptions in this paper that the working groups took this theme quite seriously, in the sense that the assigned estimates of uncertainty are many times larger than those coming strictly from the weighted averages.

Realization experiments for $K_{J\text{-}90}$ and $R_{K\text{-}90}$ can be separated into two classes: "indirect" are those which arrive at the value through the formulas for K_J and R_K. "Direct" realizations are those which compare the Josephson voltage to the SI volt, or the quantized Hall resistance to the SI ohm. Clearly the direct realizations are preferable, because they don't require extra assumptions about the validity of particular formulas.

D. Realization of the ohm, and $R_{K\text{-}90}$

The quantized Hall effect refers to the measurement of current and voltage in a bar of high-mobility (typically GaAs/AlGaAs heterostructure) semiconducting Hall bar, under a large (of order 10 T) magnetic flux density.[12] At low temperatures, the ratio of transverse voltage to longitudinal current becomes quantized for some ranges of magnetic field, with the value

$$V_{\text{trans}}/I_{\text{long}} \equiv R_K = h/ie^2 (\approx 26\,000\ \Omega \text{ for integer } i=1).$$

In the realization of the SI ohm through the value of $R_{K\text{-}90}$, the distinction between direct and indirect experiments had an important effect:[11] The direct measurements (seven independent ones) were all comparisons of the QHR standard to a SI resistance based on a calculable capacitor experiment, at seven different national labs. These comparisons were performed via long transfer chains from capacitance at frequency f to reactance at frequency f to dc resistance:

$$R = 1/(2\pi f C).$$

Then, by comparing this resistance to the QHR standard, a direct measurement of the resistance of the QHR standard in terms of the SI ohm was obtained.

The indirect measurements were all based on measurements of the fine structure constant α, using the relation

$$R_K = \mu_0 c/2\alpha.$$

Here, since μ_0 and c are both defined constants (no uncertainty) only α is needed to evaluate R_K. The best value for α arose from experimental atomic physics measurements, together with numerical perturbation method calculations of quantum electrodynamics, with α the expansion parameter. This resulted in a relative standard uncertainty of 8×10^{-9} from the indirect measurements for R_K, which for a straight weighted average would result in this measurement dominating (80%) the final answer; here the weighting is $1/\sigma_i^2$, where σ_i was the reported (one standard deviation) uncertainty for each reported result.

The working group decided, instead, to do a simple arithmetic average of this result with the weighted average of the direct measurements. Thus the final assigned value $R_{K\text{-}90}$ is one half due to the α measurement, and one half due to the QHR and calculable capacitor measurements, of which the NIST value was weighted 60%. In line with the guiding principle described above, the assigned uncertainty is quite large, being 2.5 times the difference between the direct and indirect results.

E. Realization of the volt and $K_{J\text{-}90}$

The Josephson effect produces the voltage which develops across a superconducting tunnel junction, when exposed to radio-frequency radiation. The voltage takes on a quantized value, dependent only on fundamental constants and the frequency f, $V = nhf/2e \equiv f/K_J$. At $f\approx 80$ GHz and integer $n=1$, this yields a voltage of about 0.15 mV. Thus, in practice, an array of roughly 3000 junctions is used to provide about 1 V (many junctions operate with $n>1$).

For the assignment of the value for $K_{J\text{-}90}$, both direct and indirect methods were considered, including a capacitor volt balance [direct], several moving-coil balances [direct], and measurements of various fundamental constants including R_∞ (Rydberg constant) [indirect], N_A (Avogadro's constant) [indirect], and γ'_p (proton gyromagnetic ratio). However, the distinction between direct and indirect turned out to be unimportant: One direct measurement dominated the weighted average of eight experiments (ten were considered but two rejected because they were outliers).

This dominant experiment (comprising 80% of the weight) was the moving-coil balance (also called the watt balance) at NPL (the British standards lab), together with their calculable capacitor. The NPL moving-coil balance experiment contains two steps:[4] In the first, a coil of length L carrying a current I is placed in an orthogonal magnetic flux density B, and the force is measured via the gravitational force on a mass m:

$$I\int B\,\mathrm{d}L=mg.$$

In the second step, the induced voltage in the coil is measured as it moves at speed v:

$$V=v\int B\,\mathrm{d}L.$$

Thus

$$VI=mgv. \tag{1}$$

We note that by doing this two-phase experiment, the $\int B\mathrm{d}L$ term is eliminated, so that it is unnecessary to measure the exact shape of the loop and the exact profile of the magnetic field. Equation (1) provides an equivalence between power measured electrically and mechanically, and thus is a realization of electrical power in terms of the SI watt (thus the name "watt balance").

To realize the SI volt from the SI watt, it was then necessary to depend on the calculable capacitor to provide a resistance in terms of the SI ohm, as described in the previous section:

$$R=1/(2\pi fC).$$

Thus finally a voltage measured in terms of the SI volt was obtained:

$$V=\sqrt{mgv/(2\pi fC)}.$$

This voltage was compared to the output of the Josephson voltage standard, and thus the value of $K_{\mathrm{J}\text{-}90}$ was obtained. As in the case of $R_{\mathrm{K}\text{-}90}$, the assigned uncertainty was much larger than the simple weighted uncertainty.

F. Example of commercial calibration

The true impact of the practice of realizations and representations of a unit can only be understood in the context of an actual dissemination from primary standard to commercial product. There are several motivations for companies to use calibrations (directly or indirectly) from national standards laboratories. The main one is that traceability to the national laboratories' standards in principle ensures that the output of measurement equipment will agree with other measurements; this is particularly important in products for international use. In fact, over time more equipment purchasers are insisting, as part of the purchase, on clear traceability as part of the purchase contract.

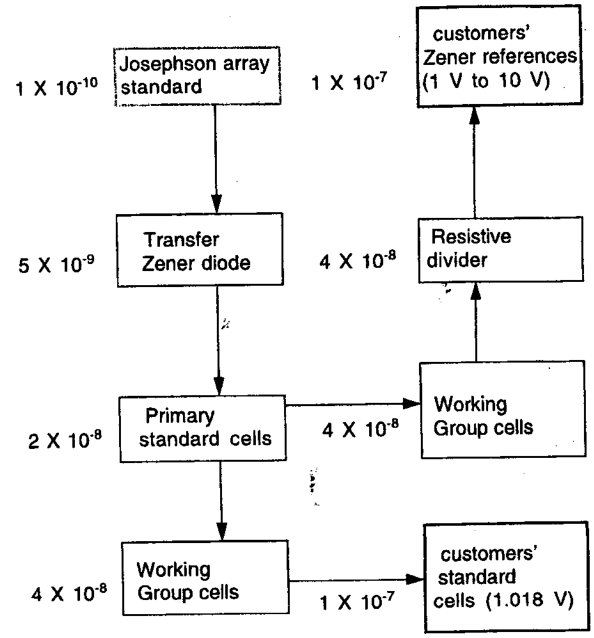

Fig. 4. Schematic framework for the part of the voltage dissemination chain which is performed at NIST. Fractional amounts to the left of each box are the relative standard uncertainties; note that these always increase as one moves along the chain.

As an example of this dissemination, I will describe in outline the flywheel for one type of voltage calibration. See Fig. 4 for a schematic outline of the part of the flywheel performed at NIST. The dissemination starts at NIST with a periodic comparison (or "transfer") of a 1-V Josephson junction array to a set of three Zener standards (about once per month). The outputs of these three are then immediately transferred (within an hour or so) to a larger set of standard (electrochemical) cells, both to check performance to date, as well as to predict the voltage drift for the next month. Although undisturbed standard cells are more stable than the Zeners, it has been determined that they can be perturbed by the connection to the Josephson junction array, which is why the three Zeners are used as buffers. The outputs of the set of standard cells are also intercompared on a more frequent basis (approximately weekly).

The values of these standard cells are transferred daily to two "working groups" of standard cells; one set of working group cells are connected to outside customers' traveling standard cells for calibration. Zener standards are more often sent by customers than the standard cells, and so the other set of working group cells are used, with a resistive voltage divider, to calibrate the customers' Zeners. After calibration and return to the customer, the traveling standard's value is transferred to the (nonmobile) primary standard of the customer's internal calibration laboratory. Then another flywheel occurs within the internal laboratory, with the ultimate working standards used to calibrate high-performance test equipment, such as many-digit voltmeters.

In this long chain from a single national standard to millions of consumer products on the factory floor, the uncertainty always gets larger (worse), or at best remains unchanged, at each transfer. Commercial calibrations of dc voltage standards at NIST are routinely performed with a stated standard uncertainty of about 0.3×10^{-6} V for 1 V, with smaller uncertainties available. Each step of the transfer within NIST, and beyond in the company's internal calibration laboratory, should be analyzed to determine the increase of uncertainty.

I should note that one important result of the quantum standards is that they allow other organizations besides national standards laboratories to maintain primary standards with low uncertainty. Thus about ten other government and commercial laboratories in the U.S. maintain Josephson standards, giving them nominally the same low uncertainty internally. Since NIST maintains the legal U.S. standard volt, these customers must still do periodic comparisons, but they can do them less often. This substantially decreases the uncertainties achievable within the customer's internal laboratory and has made possible new commercial products not otherwise available.[13] Over the past five years, NIST's voltage calibration business has decreased in total volume; this is due to several technical advances, and an important one is indeed the availability of the Josephson standards.

IV. IMPLICATIONS OF NEW EXPERIMENTS

A. Electronic kilogram

Up until now, the quantum standards for voltage and resistance have only been used for representation, because the coherency of the SI units requires realization based on the mechanical units, including the kilogram. As mentioned earlier, the kilogram is still an "artifact," a Pt–Ir weight stored at the BIPM.

The problems with this artifact, the international prototype of the kilogram, include an apparent drift of order 50 μg (relative drift 5×10^{-8}) over the last century. It has been proposed by many workers[14] that the kg artifact could be replaced by a more fundamental standard. As discussed in Sec. II, defining mass in terms of "fundamental" units would allow standards that depend on mass (such as the electrical units) to also be defined in terms of fundamental units (such as the quantum standards), thus potentially allowing the full convenience and reproducibility of the JV and QHR standards to be reflected in SI units.

All of the possibilities for a new mass standard can be thought of as reflecting on the value of Avogadro's number N_A;[14] one example of a direct measure involves quantifying the density and volume, lattice constant, etc., of single massive crystals of silicon. Specializing to electrical experiments, there are several indirect methods to replace the kilogram; the electrical experiment with the lowest uncertainty, and thus the biggest potential impact on the kg standard, involves redefining N_A through Planck's constant h. The measurement of h uses a moving-coil balance in conjunction with the voltage from a JV standard, and the resistance from a QHR standard. There are several groups which have been working for years to develop moving-coil balances with relative standard uncertainties (for power) of about 10^{-8} (for power levels of approximately 10 mW, this means a standard uncertainty of roughly 0.1 nW), which is a rule of thumb for the level at which monitoring of the kilogram (artifact) would be useful. The current best reported value is still that from NPL used in the 1990 determination of $K_{J\text{-}90}$, which had a relative standard uncertainty of about 1.4×10^{-7}.

If the present experiments (primarily being pursued at NIST and NPL) succeed in monitoring the kilogram artifact in terms of fundamental constants, the ultimate goal would be to replace the definition of the kilogram. As mentioned above, this would be accomplished by using, e.g., the JV and QHR standards, in conjunction with the moving-coil balance, to *define the kilogram based on electrical standards*. This idea is the origin of the term "electronic kilogram." Such a major change in the SI would be approached very cautiously, and would require a vote by the CGPM. Thus, we don't anticipate this change occurring soon.

B. SET (single electron tunneling) devices

Finally, I wish to describe the possible impact on dissemination of the SI electrical units of a new class of low-temperature quantum electrical device, the SET pump. These changes include use as a fundamental standard of current, or, by charging a capacitor, as a standard for capacitance. They may also make possible a new measurement of α, the fine-structure constant. Also, in combination with the JV and QHR standards, the SET pump can provide a high-accuracy test of Ohm's law (the "metrology triangle" formed by voltage, current, and resistance).

SET devices[15] depend on ultrasmall tunnel junctions with very small total capacitances C (of order 10^{-15} F or less). Their operation is based on the Coulomb blockade,[16] in which the *capacitor charging energy for a single electron* $e^2/2C$ is large enough to become important; in that case, the detection or control of motion of single electrons becomes possible.

One particular SET device is the SET pump, in which electrons are passed through the device singly. A possible metrological application of electron pumps[17] is obvious: By locking the frequency f of passage to a standard, a fundamental current standard is formed:

$$I_{\mathrm{SET}}=ef.$$

Unfortunately, to achieve metrological accuracy, the frequency is limited to less than about 10 MHz,[17] which limits the current to about 1 pA; this value is many orders of magnitude too small to be useful as a direct current standard.

If it were possible to overcome these limitations, what would be the metrological significance of such a current standard? One result is that, in conjunction with the JV and QHR standards, this SET standard for the ampere would allow closure of this "metrology triangle" formed by Ohm's law: If we measure current by putting a voltage from the JV standard across a resistor defined by the QHR standard, then we have

$$I_{\mathrm{JV\text{-}QHR}}=V/R=(f/K_J)/R_K=(hf/2e)/(h/e^2)=ef/2. \quad (2)$$

Thus, by comparing I_{SET} and $2I_{\mathrm{JV\text{-}QHR}}$, we are in essence checking the validity of the physical laws and formulas for K_J and R_K—the metrology triangle.

It has sometimes been remarked by workers in the SET field that the pump can provide a measurement of e. This proposal is somewhat misleading, because while true, SET devices cannot provide a measurement of e that is fundamentally independent of measurements from the JV and QHR standards. Why is this? As in all metrology, there are various pathways to determine e in SI units. One way relevant for this discussion would be to use the quantum electrical standards; *this pathway depends on assuming the theoretical formulas for K_J and R_K are correct.* In that case, using a moving-coil balance (refer back to Sec. III E), and measuring the voltage and resistances used in the moving-coil balance with respect to the JV and QHR standards, we have

$$V = A_1(f/K_J), \qquad R = A_2 R_K,$$

where A_1 and A_2 are experimentally determined constants of proportionality. Then, using Eq. (1), we would obtain

$$mgv = V^2/R = A_3(f/K_J)^2/R_K = A_3(hf/2e)^2/(h/e^2) = A_3 hf^2/4, \qquad (3)$$

or

$$h = 4mgv/A_3 f^2,$$

Here $A_3 \equiv A_1^2/A_2$ is another experimentally determined (unitless) quantity. With all the quantities on the right-hand side of this last equation being mechanical, this produces a measurement of h in SI units (similarly for h/e, e, etc.)

We could instead use the SET pump current, together with either the output of a JV or QHR standard, through a formula analogous to (3), to measure e or h. However, we can see from Eq. (2) that this would not yield a value which is independent, because the same information is being used in both cases [frequency standard plus various mechanical quantities in Eq. (1)]. In essence, any two of the three legs of the metrology triangle are sufficient—the third is not independent.

Another metrological implication of SET devices is that they can provide a practical standard for capacitance, or an independent measurement of α.[18] The basic idea of this experiment is simple: Using 1 pA from a SET pump, in 1 s we can charge a 1-pF capacitor to 1 V (which we can measure with metrological accuracy). Since this does not produce a measurement of the SI farad, it is not a realization; however, since it may be simpler to perform than the calculable capacitor, it may form the basis of a more transportable practical standard or representation for capacitance, similar to the JV and QHR standards. In addition, by measuring the capacitor with respect to the calculable capacitor, and the voltage with respect to the JV standard, we can perform a high-accuracy measurement of α.[18] This has strong implications for metrology, and in particular for high-precision calculations using QED theory.[19]

To summarize: The impact of SET devices on the SI is not that they can provide a fundamentally different measurement of any fundamental constant. Rather, the impacts are that (1) through Ohm's law, they could strengthen the assumptions underlying the use of the formulas for K_J and R_K, and (2) they allow for a practical capacitance standard or measurement of α, by charging a capacitor with a countable number of electrons.

V. CONCLUSIONS

Although not usually considered in any detail in standard undergraduate or graduate physics courses, the professional study of the physical units and constants affords a fascinating view of the fundamental laws, involving tests of their ultimate validity. Because metrology experiments are in most cases conceptually quite simple, they provide an interesting way for teachers to tie together many of the laws. In addition, because they are used in the dissemination from primary standard to every measurement of voltage, time, mass, etc., they demonstrate the way in which the everyday application of these basic laws of physics underlies the entire economy.

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